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Turbulent mass transfer in small radius ratio annuli

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Mass transfer in annuli for both fully developed laminar and turbulent flow conditions has been studied with respect to available experimental data. It is shown that prediction of the Sherwood number for the inner annular wall based on the hypothesis of coincidence of the zero shear stress position for laminar and turbulent flows leads to serious error in the case of small radius ratio. Also it is shown that in contrast with plain tubes the curvature in small radius ratio annuli should be taken into account for the case of small Reynolds numbers. In consequence, the well-known Leveque equation can be used for the calculation of the mass transfer coefficient in annuli only under certain conditions. Possibilities of electrodiffusion diagnostics for the precise determination of the zero shear stress position in annuli are discussed.

List of symbols

A	cross-section flow area (m^2)
$a = r_1/r_2$	annular radius ratio (—)
$\bar{c}$, c', c_0	mean fluctuation and bulk concentration ($\mathrm{mol\,m^{-3}}$)
D	molecular diffusivity ($\mathrm{m^2\,s^{-1}}$)
d_h	hydraulic diameter (m)
f, f_1, f_2	overall, inner and outer wall friction factors (—)
$f_\tau = \tau_1/\mu$	near wall velocity gradient ($\mathrm{s^{-1}}$)
$\bar{\imath}$	pressure drop per unit of length ($\mathrm{Pa\,m^{-1}}$)
K_L	average mass transfer coefficient ($\mathrm{m\,s^{-1}}$)
$k = r_0/r_{0,L}$	ratio of zero shear stress position in turbulent and laminar flows (—)
L	mass transfer surface length (m)
L_D	diffusion 'leading edge' length (m)
L_{ent}	diffusion entrance length (m)
P_W	wetted perimeter (m)
$Re = U_{av}d_h/\nu$	Reynolds number (—)
r	radial distance from conduit axis (m)
r_0, $r_{0,L}$	radial distance of zero shear stress position in turbulent and laminar flows (m)
r_1, r_2	radius of inner and outer annular cylinders (m)
$Sc = \nu/D$	molecular Schmidt number (—)
$Sh = K_L d_h/D$	Sherwood number (—)
U_{av}	average liquid velocity ($\mathrm{m\,s^{-1}}$)
u, u'	mean and fluctuation axial velocity ($\mathrm{m\,s^{-1}}$)
v, v'	mean and fluctuation radial velocity ($\mathrm{m\,s^{-1}}$)
$y = r - r_1$	distance from the inner wall (m)
$y_\tau = \nu(\rho/\tau_1)^{1/2}$	dynamic length (m)
z	distance in direction of the flow (m)

Greek symbols

δ_D	diffusion layer thickness (m)
μ	dynamic viscosity (Pa s)
ν	kinematic viscosity ($\mathrm{m^2\,s^{-1}}$)
ρ	density ($\mathrm{kg\,m^{-3}}$)
τ	shear stress (Pa)
τ_W	wall shear stress for tube and plate channel (Pa)
τ_1, τ_2	wall shear stress for inner and outer annular cylinders (Pa)
Φ	Geometrical factor with respect to k-function (—)
Φ_R, Φ_K	geometrical factor with respect to Rothfus or Kays–Leung equations (—)
λ	ratio of radial distance of zero shear stress position to outer radius in laminar flow (—)

1. Introduction

The concentric annular geometry has many engineering applications, for example in heat exchanger and nuclear reactor design. Also, mass transfer to fluids flowing through annuli is frequently encountered in industrial processes: electrochemical reactors, condensers, transpiration and film cooling of ducts. As such, flow and mass transfer in an annular geometry have been studied extensively by many workers. Nevertheless, even in the simplified case of fully developed flow in smooth annuli, previous

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papers [1–7] give an incomplete account of the principles of mass transfer from flowing solution, especially for the case of turbulent flow.

The key problem in turbulent flow through smooth annuli is the determination of the position of the zero shear stress plane and hence, the wall shear stress at the inner and outer cylinders, respectively. Previously the mass transfer problem in annuli has been studied by assuming the coincidence of zero shear stress position for laminar and turbulent flows. Yet, this assumption cannot be applied in the case of annuli with small radius ratio, as it has been shown elsewhere [8–20]. So, the prediction of the friction factor for the inner cylinder, based on the classical works of Rothfus [21,22] is not sufficient for the calculation of the Sherwood number, *Sh*, especially for turbulent flow in annuli with very small radius ratio.

The case where the inner cylinder radius is small with respect to the outer one should be studied for comparison of the results obtained in annulus, channel and tube. The simple solution developed by Leveque for the heat-transfer coefficient in laminar flow past a flat plate can be expressed in mass transfer terms and without any problem applied to the case of tubes. For the case of annuli this procedure is correct only if the definite relation between the Reynolds number, *Re*, the length of the active part of the surface, *L*, and the outer and inner annulus radii is verified. The concrete form of this relation will be discussed.

2. Convective diffusion equation for the annular flow cell

The basic differential equation of turbulent diffusion in polar cylindrical coordinates with axial symmetry may be written:

$$\bar{u}\frac{\partial \bar{c}}{\partial z} + \bar{v}\frac{\partial \bar{c}}{\partial r} + \frac{\partial(\overline{c'u'})}{\partial z} + \frac{1}{r}\frac{\partial(r\overline{c'v'})}{\partial r} = D \times \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \bar{c}}{\partial r}\right) + \frac{\partial^2 \bar{c}}{\partial z^2}\right] \tag{1}$$

For fully developed hydrodynamic conditions, we have

$$\bar{v} = \bar{w} = 0; \quad \bar{u} = \bar{u}(r)$$

For this case Equation 1 takes the form

$$\bar{u}\frac{\partial \bar{c}}{\partial z} + \frac{\partial(\overline{c'u'})}{\partial z} + \frac{1}{r}\frac{\partial(r\overline{c'v'})}{\partial r} = D \times \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \bar{c}}{\partial r}\right) + \frac{\partial^2 \bar{c}}{\partial z^2}\right] \tag{2}$$

We consider the case where the active part of the surface (a cathode, for example) forms part of the inner wall of the annular cell. The mass transfer surface length L (i.e. the electrode length) is assumed to be small in comparison with the diffusion entrance length L_{ent}. For turbulent flow the diffusion entrance length can be defined by the length of the zone where a diffusion layer of a constant thickness is formed. Within this entrance zone mass transfer to the surface occurs through the mean axial velocity and molecular diffusion. Turbulent fluctuations do not play any considerable role and the diffusion layer thickness increases downstream with the flow. In contrast, outside the entrance zone the mass transfer to the surface is due to the normal velocity pulsations as well as molecular diffusion and the diffusion layer has a constant thickness. So, for the case when $L \ll L_{\mathrm{ent}}$ both turbulent terms on the left-hand side of Equation 2 can be neglected.

On the other hand, we consider that the cathode length L is large in comparison with the diffusion 'leading edge' L_{D}, which is the length of the zone where axial molecular diffusion is important. Outside this 'leading edge' axial molecular diffusion has no influence and, hence, the last term on the right-hand side of Equation 2 can be neglected.

The criteria $L \ll L_{\mathrm{ent}}$ and $L \gg L_{\mathrm{D}}$ have been analysed in a number of papers [23–28]. To obtain the concrete form of these criteria one can use the estimations proposed by Hanratty [28]:

$$0.5y_\tau < L < 700y_\tau \tag{3}$$

where y_τ is the dynamic length. Of course, the diffusion entrance length, as well as the length of the diffusion 'leading edge', depend on the molecular Schmidt number, *Sc* [27]:

$$L_{\mathrm{D}} \sim Sc^{-1/2}y_\tau$$
$$L_{\mathrm{ent}} \sim Sc^{1/4}\epsilon_{yy}^{-3/2}(T_{yy}f_\tau)^{-3/4}y_\tau \tag{4}$$

where ϵ_{yy} is the turbulence intensity of the normal velocity on the external boundary of the viscous sublayer ($y = 5y_\tau$), T_{yy} is the correlation time of the normal velocity fluctuations within the viscous sublayer and f_τ is the near wall mean velocity gradient.

So, the numerical constants in Equation 3 correspond to typical conditions of mass transfer experiments with $Sc \sim 1000$. As an example, for annuli with moderate radius ratio, the diffusion entrance length L_{ent} is about 15 cm for $Re \simeq 10^4$ and $d_{\mathrm{h}} \simeq 10$ cm. Using the above-mentioned assumptions Equation 2 simplifies to

$$\bar{u}(y)\frac{\partial \bar{c}}{\partial z} = \frac{D}{r_1 + y}\frac{\partial}{\partial y}\left[(r_1 + y)\frac{\partial \bar{c}}{\partial y}\right] \tag{5}$$

with the following boundary conditions

$$\left.\begin{array}{l} c \longrightarrow c_0 \text{ as } y \longrightarrow \infty, \quad \text{or } z \longrightarrow 0 \\ c \longrightarrow 0 \text{ as } y \longrightarrow 0 \end{array}\right\} \tag{6}$$

The beginning of the z-axis coincides with the beginning of the mass transfer surface, and $y = r - r_1$ denotes the distance from the inner wall.

The form of Equation 5 and the boundary conditions (Equation 6) are absolutely the same for laminar and for turbulent annular flows. So, all the differences between laminar and turbulent mass transfer coefficients will be connected with the differences of the velocity profiles.

Additional simplifications can be made because of the large magnitude of the molecular Schmidt number for the mass transfer problem. The diffusion boundary layer is very thin, so the velocity near the inner wall can be conveniently expressed in terms of the wall shear stress:

$$\bar{u}(y) = (\tau_1/\mu)y \tag{7}$$

Usually [1–7] the solution of the mass transfer problem in tubes and annuli is obtained from the Leveque equation:

$$\frac{\tau_1}{\mu} y \frac{\partial \bar{c}}{\partial z} = D \frac{\partial^2 \bar{c}}{\partial y^2} \tag{8}$$

It should be emphasized that the transition from Equation 5 to Equation 8 is connected with modification of the form of the right-hand side term in this equation. This modification is correct only if the diffusion layer thickness δ_D is small in comparison with the radius of the inner cylinder r_1:

$$\delta_D \ll r_1 \tag{9}$$

To obtain a concrete form of the Estimation 9 the Leveque solution for the concentration profile can be used as well as the well known velocity distribution in annuli under laminar flow conditions [30]. As a result Expression 9 for laminar flows in the case of small annular ratio is transformed to

$$L \ll L_{max} \tag{10}$$

where

$$L_{max} = Sc\ Re \frac{a^2}{\ln(1/a)} r_2 \qquad (a \ll 1) \tag{11}$$

gives the estimation for the maximum electrode length and the Leveque equation may still be used to predict the mass transfer coefficient for the inner annular wall.

For rather long electrodes ($L \gtrsim L_{max}$) Equation 5 with the linear velocity profile (Equation 7) should be used to calculate the mass transfer coefficient for the inner cylinder.

For tube flow the estimation of the maximum electrode length L_{max} also takes the form of Equation 11; if, in this equation, the geometrical factor $a^2/\ln(1/a)$ is transformed to unity and the outer radius of the annulus r_2 to the tube radius. So, for flow in tubes, the maximum electrode length is very large and the curvature does not play a significant role.

Equation 11 demonstrates that, in contrast with flow in tubes, the curvature in annuli has an influence if both the annular ratio and the Reynolds number are rather small. For example, if the annular ratio, a, is equal to 0.01 and the Reynolds number is about or smaller than 10, the curvature of the geometry should be taken into account for the inner mass transfer surfaces with length comparable with or larger than the outer annular radius.

For turbulent flow in annuli the estimation of the maximum length, L_{max}, takes the form:

$$L_{max} = \frac{1}{200} Re^{7/4} Sc \frac{a^2}{\ln(1/a)} r_2 \qquad (a \ll 1) \tag{12}$$

and the curvature does not play any important role because of the large magnitude of the Reynolds number.

The main conclusion is that the well-known Leveque solution for the mass transfer coefficient [29]

$$K_L = 0.807 D(\tau_1/\mu D L)^{1/3} \tag{13}$$

can be used in annuli, as well as in plain tubes, in all cases where the mass transfer surface length L is small in comparison with the maximum length L_{max} (Equations 11–12). In the opposite case ($L \gtrsim L_{max}$), which takes place for annuli with small radius ratio at low Reynolds numbers, the Leveque solution (Equation 13) cannot be used. For this case the expression for the Sherwood number can be obtained by solving Equation 5.

3. Hydrodynamic characteristics of the flow in annuli

3.1. *Friction factor in annuli*

Overall friction factors, f, for tubes, channels, passages and annuli may be calculated from pressure drop measurements, using

$$f = \frac{\bar{\tau}}{2\rho U_{av}^2/d_h} \tag{14}$$

and can be presented in terms of the hydraulic diameter $d_h = 4A/P_w$, where A is cross-sectional flow area and P_w is wetted perimeter. For concentric annuli the hydraulic diameter, d_h, is equal to $2(r_2 - r_1)$.

For circular tubes and parallel plate channels the pressure drop for fully developed hydrodynamic conditions is directly connected to the wall shear stress τ_w:

$$\tau_w = \bar{\tau} A/P_w \tag{15}$$

so the friction factor can also be written in terms of the wall shear stress as follows:

$$f = \frac{2\tau_w}{\rho U_{av}^2} \tag{16}$$

For annuli the wall shear stresses at outer, τ_2, and inner, τ_1, cylinders are different, so Equation 15 is modified to

$$\bar{\tau} = 2\frac{\tau_2 r_2 + \tau_1 r_1}{r_2^2 - r_1^2} \tag{17}$$

Hence, two different friction factors should be introduced, as was done for the first time by

Rothfus [21],

$$f_1 = \frac{2\tau_1}{\rho U_{av}^2} \text{ and } f_2 = \frac{2\tau_2}{\rho U_{av}^2} \tag{18}$$

For smooth circular tubes and parallel plate channels the dependence of the overall friction factor on the Reynolds number is well established and can be described by the classical Blasius law:

$$f = 0.079 Re^{-1/4} \tag{19}$$

or for large values of the Reynolds number, $Re > 10^5$, by Nikuradse's law [34]:

$$f^{-1/2} = 4.0 \log(Re f^{1/2}) - 0.4 \tag{20}$$

The deviation of the data of different authors is within 10% accuracy. This deviation is connected with instrumental errors as well as with differences in the experimental conditions (different range of the Reynolds number, different length of the entrance section, etc.).

Numerous attempts have been made to establish the dependence of the overall friction factor, f, on Reynolds number in annuli. In some early works [21, 31–33] a significant difference between overall friction factor in plain tubes and concentric annuli was found. But there was evidently little agreement between these investigations: opposite conclusions had been drawn about the dependence of the overall friction factor on the radius ratio and the Reynolds number. Obviously the conclusion that the friction factors in tubes and annuli are significantly different is connected with instrumental errors.

More careful and extensive studies [8–10] showed that overall friction factors for annuli are very near to those in plain tubes. Probably the overall friction factor in annuli increases slightly (about 10% in comparison with plain tubes) with increase in the radius ratio. This corresponds with the results of [11] according to which the pressure drop coefficients of parallel plate channels are about 5% higher than the circular tube values.

So, the available experimental data show that for plain tubes, parallel plate channels and concentric annuli the same Blasius (Equation 19) or Nikuradse (Equation 20) laws can be used to predict the overall friction factor.

3.2. *Correlation between the wall shear stress and the position of the zero shear stress*

Prediction of the friction factors for inner f_1 and outer f_2 annular cylinders is a more complicated problem. This problem can be reduced to the prediction of the zero shear plane position.

Let us obtain the equation for the position of zero shear stress in terms of the wall shear stress for the case of fully developed turbulent flow in smooth annuli. The equation for the time-averaged axial fluid velocity distribution for this case takes the form:

$$\bar{\imath} = \frac{\rho}{r}\frac{d}{dr}\left[r\left(\nu\frac{d\bar{u}}{dr} - \overline{u'v'}\right)\right] \tag{21}$$

By integrating Equation 21 with respect to r and applying the boundary conditions at the inner and outer walls, the equation for the shear stress is obtained:

$$\tau(r) = -\rho\nu\frac{d\bar{u}}{dr} + \rho\overline{u'v'} = -\frac{\bar{\imath}}{2}\left(r - \frac{r_0^2}{r}\right) \tag{22}$$

as well as two forms of the equation for the position of the zero shear plane r_0:

$$\tau_1 = -\frac{\bar{\imath}}{2}\left(r_1 - \frac{r_0^2}{r_1}\right) \tag{23}$$

$$\tau_2 = -\frac{\bar{\imath}}{2}\left(r_2 - \frac{r_0^2}{r_2}\right) \tag{24}$$

Combining Equations 23 and 24 and taking into account Expression 17 for the pressure drop, the symmetrical form of the equation for the zero shear stress position can be obtained. Thus,

$$r_0 = \left[\frac{r_1 r_2(\tau_2 r_1 + \tau_1 r_2)}{r_2\tau_2 + r_1\tau_1}\right]^{1/2} = \left[\frac{2}{\bar{\imath}}\frac{r_1 r_2(\tau_2 r_1 + \tau_1 r_2)}{(r_2^2 - r_1^2)}\right]^{1/2} \tag{25}$$

Equations 17, 23 and 24 can also be obtained by consideration of the force balance between the wall shear stress and the pressure drop. For the case of small radius ratio Equation 25 simplifies to

$$r_0 = \left(\frac{2\tau_1 r_1}{\bar{\imath}}\right)^{1/2} \tag{26}$$

as $a \ll 1$.

Equation 23 or its limiting case Equation 26 allows prediction of the wall shear stress at the inner cylinder if the position of the zero shear plane is known.

3.3. *Location of the position of the zero shear plane*

In laminar flow the position of zero shear stress coincides with the position of the maximum velocity and is given by Lamb's Equation 30:

$$r_{0,L} = r_2\left[\frac{1 - a^2}{2\ln(1/a)}\right]^{1/2} \tag{27}$$

Rothfus [21, 22] and later Knudsen [35] supposed a coincidence between the position of the zero shear plane in laminar and in fully developed turbulent flows. This supposition permitted them to predict the inner wall friction factor and, hence, the wall shear stress at the inner cylinder. That is,

$$f_1 = 0.079\ Re^{-1/4}\Phi_R(a) \tag{28}$$

where the geometric factor $\Phi_R(a)$ is equal to

$$\Phi_R(a) = \left(\frac{1-a}{1-\lambda^2}\right)^{1/4} \frac{\lambda^2 - a^2}{a(1-\lambda^2)} \tag{29}$$

where $\lambda = r_{0,L}/r_2$.

The position of the zero shear stress both for laminar and turbulent fully developed flow in annuli can be predicted by the same Equation 27 only for the case of large and moderate annular ratii. Yet, this assumption does not hold in the case of small annular ratii when the velocity profile is very asymmetric. This fact was established in the experimental works of Lorenz [36], Brighton and Jones [8] and confirmed by other experiments [9–15, 37, 38].

Also, a number of attempts have been made to obtain the correct equation for the position of the zero shear stress, r_0, in fully developed turbulent flow. These attempts were based on a statistical analysis of experimental results for the case of small annular radius ratio as well as on theoretical investigations [16–20].

Generally, authors do not distinguish the position of the maximum velocity and the position of the zero shear stress. For example, Kays and Leung [16] correlated previous experimental data both for the zero shear stress and maximum velocity positions by the equation

$$\frac{r_0 - r_1}{r_2 - r_0} = \left(\frac{r_1}{r_2}\right)^n \tag{30}$$

with the exponent $n = 0.343$. According to Equation 30 the zero shear stress position depends on the annular radius ratio, but not on the Reynolds number.

Quarmby correlated his experimental data [9] by Equation 30 with the power $n = 0.366$. Also he obtained theoretically [17] that the power n is equal to 0.415 for high Reynolds numbers. Quarmby noted that for low Reynolds numbers the zero shear stress and the maximum velocity positions depend on Re.

Rehme [10], by analysing the literature data [8, 9, 11, 36–38], noted that the maximum velocity position in turbulent flows differs from the zero shear stress position. For high values of the Reynolds number ($Re \sim 10^5$) the maximum velocity position can be satisfactorily correlated by the Kays-Leung relation, i.e. by Equation 30 with $n = 0.343$ [10]. Rehme correlated all the previous experimental data for the zero shear stress position [11–15], as well as his own, by the same Equation 30 but with another value of the power, $n = 0.386$.

To take into account the dependence of the zero shear stress position on the Reynolds number, Rehme [10] proposed the following relations:

$$\begin{aligned} r_0/r_2 &= 0.3864 - 0.0057 \log Re \qquad &\text{for } a = 0.1 \\ r_0/r_2 &= 0.342 - 0.017 \log Re \qquad &\text{for } a = 0.04 \\ r_0/r_2 &= 0.345 - 0.02955 \log Re \qquad &\text{for } a = 0.02 \end{aligned} \tag{31}$$

Now it is definitely established that for fully developed turbulent flow in annuli with small radius ratio the position of the zero shear stress plane, r_0, significantly differs from that $r_{0,L}$ predicted by Equation 27, so that

$$r_0 = k(a, Re) r_{0,L} \tag{32}$$

where the k-function depends on the radius ratio and, probably, on the Reynolds number. For large and moderate values of the radius ratio the k-function is near unity and the difference between the zero shear stress positions in turbulent and in laminar flows is negligible. But for small values of the radius ratio this difference is important. For example, with respect to [10] for the annular ratio $a = 0.1$ the magnitude of the k-function is about 0.77; for $a = 0.04$, $k \simeq 0.65$; and for $a = 0.02$, $k \simeq 0.53$.

4. Mass transfer results

4.1. *Determination of Sherwood number for the inner annular cylinder*

The prediction of the mass transfer coefficient in turbulent flows is reduced to the prediction of the wall shear stress at the inner cylinder. Ross and Wragg [1] have made the most comprehensive contribution regarding the study of mass transfer in fully developed turbulent flows in annuli. They based their treatment on the work of Rothfus [21, 22] as well as the work of Knudsen [35], who supposed a coincidence between the position of the zero shear plane for fully developed laminar and turbulent flows. But as was noted above this assumption does not hold in the case of small annular radius ratio.

To obtain the correct form of the equation for the Sherwood number for the inner annular cylinder we use the expression for the inner wall shear stress i.e.

$$\tau_1 = \frac{\bar{\imath} r_1}{2}\left[k^2 \frac{1-a^2}{2a^2 \ln(1/a)} - 1\right] \tag{33}$$

which can be obtained by combining Equations 23, 27 and 32. By substituting Equation 33 into Equation 13 we obtain the expression for the inner wall Sherwood number in terms of the pressure gradient $\bar{\imath}$. Thus,

$$Sh_1 = 0.807\left[\frac{4\bar{\imath} r_1^4 (1-a)^3}{\mu L D a^3}\right]^{1/3}\left[k^2 \frac{1-a^2}{2a^2 \ln(1/a)} - 1\right]^{1/3} \tag{34}$$

For small values of the radius ratio $a \ll 1$ Equation 34 simplifies to

$$Sh_1 = 0.807 k^{2/3} \bar{\imath}^{1/3}\left[\frac{2r_1^4}{\mu L D a^5 \ln(1/a)}\right]^{1/3} \qquad \text{for } a \ll 1 \tag{35}$$

The transition from Equation 34 to Equation 35 is valid if both conditions $a \ll 1$ and $r_1 \ll r_0$ take place. The last condition gives the possibility of neglecting unity in the second bracket in Equation 34.

To present the Sherwood number in terms of the Reynolds number Equation 19 or 20 for overall friction factor can be used. For example, for the moderate range of Reynolds number $Re < 10^5$, the Blasius law (Equation 19) for the overall friction factor can be used and Equation 34 and 35 are reduced to

$$Sh_1 = 0.275 Sc^{1/3}(d_h/L)^{1/3} Re^{7/12}[\Phi(a)]^{1/3} \tag{36}$$

where the function $\Phi(a)$ is determined by the following equation:

$$\Phi(a) = \frac{k^2(1+a)}{2a\ln(1/a)} - \frac{a}{1-a} \tag{37}$$

or, for small values of the radius ratio, by the following equation

$$\Phi(a) = \frac{k^2}{2a\ln(1/a)} \qquad \text{for } a \ll 1 \tag{38}$$

The identification of the Φ-function with a pure geometrical factor is not correct because the k-function in Equations 37 and 38 depends on the Reynolds number as well as on the radius ratio (for example, see [9, 10]).

It is clear that the main distinction between the procedure proposed by Ross and Wragg [1] for the prediction of the Sherwood number and the present treatment is directly connected with the difference of the k-function from unity. For example, if the annular radius ratio, a, is equal to 0.1, then the magnitude of the k-function is near 0.77. Hence, the error in calculation of Sherwood number by using the Rothfus formula for the zero shear stress position is about 20%. For the annular radius ratio $a = 0.02$ the magnitude of the k-function is near 0.53 and the corresponding error in calculation of the Sherwood number will be about 50%.

It seems that Equation 36 together with the Expression 37 for the Φ-function should exactly coincide with those obtained by the Ross and Wragg procedure [1] if we equate the k-function in Equation 37 with unity. Indeed in [1] Equation 36 was obtained, but the Φ-function in this work is equal to Φ_R (Equation 29).

For the case of radius ratio $a = 0.5$ which was studied experimentally by Ross and Wragg [1] Equation 29 gives $\Phi_R = 1.295$. On the other hand Equation 37, with k-function equal to unity, gives $\Phi = 1.164$, which is 11% less than Φ_R.

This additional distinction between our treatment and the results of the work [1] is due to the form of the equation for the overall friction factor which was proposed by Rothfus [22]. In this work the Blasius law for the overall friction factor was also used. However, the Reynolds number was based on the length connected with the position of the zero shear stress but not on the hydraulic diameter, d_h. As was mentioned above the most reliable experimental data are in a good agreement with the classical form of the Blasius law (Equation 19).

4.2. *Expression of the inner wall Sherwood number for practical calculation*

Equations 34–38 give the inner wall Sherwood number in terms of the k-function. For practical calculation it is desirable to express the inner wall Sherwood number in terms of Reynolds number and geometric factor. Such an expression can be obtained by calculation of the inner wall shear stress, τ_1, (Equation 23) with respect to the Kays–Leung Equation (30) and by substituting τ_1 into the Leveque Equation (13). As a result we obtain

$$Sh_1 = 0.807 Sc^{1/3}(d_h/L)^{1/3}(f\, Re^2)^{1/3}[\Phi_k(a)]^{1/3} \tag{39}$$

where the overall friction factor f is determined by the Blasius law (Equation 19), or for large values of the Reynolds number by the Nikuradse law (Equation 20), and with the geometrical factor

$$\Phi_k(a) = \frac{a}{2(1-a)}\left[\left(\frac{a^{n-1}+1}{a^n+1}\right)^2 - 1\right] \qquad \text{for } 0.01 < a < 0.5 \tag{40}$$

With respect to the Rehme data [10] the power in Equation 40 is $n = 0.386$.

For large values of radius ratio ($a > 0.5$) Equation 39 with the geometrical factor $\Phi_k = 1$ can be used, so for this case the difference between annuli and plain tubes is negligible, at least within 5% accuracy.

Of course Equations 39 and 40 do not take into account the dependence of the zero shear stress position on the Reynolds number. To take into account this dependence Equation 31 may be used, but, more precisely, measurements of the k-function (Equation 32) should be done.

4.3. *Sherwood number for the outer annular cylinder*

The Sherwood number for the outer annular cylinder, Sh_2, can be predicted if it is taken into account that the ratio of the Sherwood numbers for the inner and outer walls is proportional to the ratio of the wall shear stress for these walls:

$$Sh_2 = |\tau_2/\tau_1|^{1/3} Sh_1 \tag{41}$$

We assume that the mass transfer surface length is the same for the inner and outer walls.

By using Equations 23, 24, 27 and 32 the following expression for the wall shear stress ratio is obtained:

$$\frac{\tau_2}{\tau_1} = \frac{a[2\ln(1/a) - (1-a^2)k^2]}{-k^2(1-a^2) + 2a^2\ln(1/a)} \tag{42}$$

which, together with Equation 41, predicts the Sherwood number for the outer annular cylinder.

For small values of the annular ratio Equation 41 simplifies to

$$\frac{\tau_2}{\tau_1} = \frac{2a\ln(1/a)}{-k^2} \quad \text{for } a \ll 1 \tag{43}$$

and the expression for the Sherwood number for the outer cylinder takes the form of Equation 36, where the geometrical factor is equal to unity (see Equations 38 and 43), so

$$Sh_2 = 0.275\ Sc^{1/3}(d_{\mathrm{h}}/L)^{1/3}Re^{7/12} \tag{44}$$

Equation 43 has absolutely the same form as the well-known equation for the Sherwood number for the case of plain tubes.

5. Possibility of electrodiffusion diagnostics for the precise determination of the zero shear stress position in annuli with small radius ratio

As discussed above the determination of the zero shear stress position is the key problem for the prediction of hydrodynamic, as well as heat and mass transfer, characteristics in annuli under fully developed turbulent flow conditions. In all the previous papers [8–15, 36–38] the determination of the zero shear stress position was done by using Pitot or Preston tubes as well as hot wire anemometers. But all these experimental techniques have one principal defect, namely, perturbation of the flow characteristics due to sensor introduction into the stream. This defect plays a considerable role for the case of annuli with small radius ratios because of the small distance between the zero shear stress plane and inner wall.

Electrodiffusion techniques which are now widely used to measure the time-average value and fluctuations of the wall shear stress in a turbulent flow [28] is free from this defect. The electrodiffusion technique is based on small mass transfer probes, mounted flush to a wall over which a fluid is flowing. An electrochemical reaction is carried out on the surface of an electrode under conditions such that the electrochemical process is controlled by the rate of mass transfer. So, the measured electric current is related to the mass transfer rate by Faraday's law and the current measurements allow determination of the Sherwood number and the wall shear stress.

To determine the zero shear stress position the working electrode can form a part of the inner annular cylinder. For annuli with small radius ratio the inner wall Sherwood number, Sh_1, can be determined by using polarographic measurements and plotted against the quantity q:

$$q = 0.807\bar{\tau}^{1/3}\left[\frac{2r_1^4}{\mu LDa^5\ln(1/a)}\right]^{1/3} \tag{45}$$

This allows (see Equation 35) the k-function to be obtained:

$$Sh_1 = k^{2/3}q \tag{46}$$

which gives the difference between the real position of the zero shear stress plane and that predicted by the Rothfus formula (Equations 27 and 31).

6. Conclusions

The present study shows that annuli with small radius ratio cannot be considered as a limiting case of plain tubes with respect to the problem of mass transfer on the inner annular wall.

In contrast, with plain tubes, two factors should be taken into account. First, due to the asymmetric character of the velocity profile, the position of the zero shear stress in annuli with small radius ratio for turbulent flow significantly differs from the laminar case. So, prediction of the Sherwood number for the inner annular wall based on the classical Rothfus hypothesis about coincidence of the zero shear stress position for laminar and turbulent flows leads to serious error in the case of small radius ratio. When the annular radius ratio, a, is equal to 0.1 the corresponding error in the prediction of the inner wall Sherwood number is about 20%. For an annulus with the radius ratio $a = 0.02$ this error is about 50%.

Equations 34–38 of this paper predict the Sherwood number for the inner annular wall in terms of the Reynolds number (or pressure drop) as well as the k-function which gives the difference between the real position of the zero shear stress plane and that predicted by the Rothfus formula (see Equations 27 and 31). For practical calculations of the inner wall Sherwood number Equation 39, with the geometrical factor Equation 40, can be recommended.

The Sherwood number for the outer annular wall has absolutely the same form as the well-known equation for the Sherwood number for the case of plain tubes, see Equation 43. The difference between annuli and plain tubes is also negligible for the inner wall Sherwood number if one deals with large values of annular ratio $a > 0.5$.

The curvature is another factor which should be taken into account in annuli in contrast with plain tubes. Because of very high (infinite in the limiting case $a \longrightarrow 0$) magnitude of the inner wall shear stress the well-known Leveque equation can be applied for the prediction of the inner wall Sherwood number only under certain conditions. In other words the Leveque equation may be used only if the length of the mass transfer surface is small enough in comparison with the maximum length defined by Equations 11 and 12. The curvature of the geometry plays an essential role for annuli with small radius ratio in the case of low Reynolds numbers.

The present study shows that the position of zero shear stress in annuli can be very precisely determined by using electrodiffusion diagnostics (see Equation 43), which has definite advantage in comparison with Pitot or Preston tubes and hot wire anemometers.

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